

1. First Variant of the Model. Numerical investigation of hydrodynamic resistance and temperature conditions in different heat exchangers reduces to the determination of the mean velocity and temperature fields in turbulent flows in channels with complex shapes. Considerable attention has been paid during the last few decades to the development of methods to solve similar problems.

A theoretical model for turbulent transfer was developed in [1, 2] which could approximate all six components of the symmetric turbulent shear stress tensor and three components of the turbulent heat flux vector in the entire flow field right up to the very walls. This model was widely used to compute turbulent flows and heat transfer in steady flows in different channels (see [3]).

The model [1, 2] is further developed in [4, 5].

With the modifications introduced in [4, 5], the turbulence model appears as follows.

Turbulent flow is considered as the result of the superposition of random, nonstationary eddies on a certain mean flow. Inside each random and rapidly disappearing eddy with a certain transverse dimension $2l$, fluid portions with transverse dimension l (mole) are transferred over a distance of the order l .

The concept of integral length scale for turbulence $L(M)$ is introduced to reflect the characteristic transverse dimension of nonstationary eddies in the neighborhood of the variable point M of the flow and transferred in this eddy by the mole:

$$d = \beta L,$$

where d is the "diameter" of moles; β is an empirical constant.

Furthermore, the concept of directed turbulence length scale $L_s(m)$ is introduced to indicate the characteristic mixing length of the moles from the neighborhood of the point M in the direction s .

To the first approximation, the integral length scale $L(M)$ is considered to be associated only with the characteristic distance from the point M to the channel wall and determined by the equation

$$\frac{1}{L} = \frac{1}{\pi} \int_{\Omega} \frac{1}{l} d\Omega,$$

where $l(\Omega)$ is the distance from the point M to the channel wall in the direction Ω .

The directed turbulent length scale is determined by the equation

$$\frac{1}{L_s} = \frac{3}{2} \frac{1}{\pi} \int_{\Omega} \frac{1}{l} |\cos(l, s)| d\Omega.$$

A weighting function $\varphi(M, M_0)$ is introduced to reflect the probability of the mole passing through the given point M_0 from the neighborhood of the variable point M of the surrounding flow field. The structure of the weighting function was corrected during the development of the model in [2, 5]. In [5] the function was used in the form

$$\varphi(M, M_0) = \frac{3}{\pi \lambda_{s0}^3} \left(1 - \frac{s}{\lambda_{s0}}\right), \quad \lambda_{s0} = \alpha L_{s0}, \quad s < \lambda_{s0},$$

where s is the distance between the points M_0 and M ; λ_{s0} is the value of the order of two characteristic mixing lengths of the mole in the direction s in the neighborhood of the point

M_0 ; α is an empirical constant.

The model is based on the following quantitative hypotheses.

1. The magnitude of the characteristic velocity of the mole moving in the neighborhood of the point M in the direction s is proportional to the magnitude of the deformation $|\partial V/\partial n|$ of the mean velocity field at the point M and to the directed length scale L_s at this point:

$$V_s'^2 = \begin{cases} 0, & \text{if } \gamma_* < \omega, \\ \mu^2 L_s^2 \left| \frac{\partial V}{\partial n} \right|^2, & \text{if } \gamma_* \geq \omega, \end{cases} \quad \gamma_* \equiv \frac{L^2}{\nu} \left| \frac{\partial V}{\partial n} \right|. \quad (1.1)$$

2. The interaction of the moving mole with the surrounding fluid is described by the equations:

$$du_i^* = \frac{3}{R} A_1 (\bar{u}_i - u_i^*) dt, \quad dT^* = \frac{3}{R} A_2 (\bar{T} - T^*) dt, \quad (1.2)$$

where u_i^* is the velocity component of the mole along the axis x_i ; T^* , temperature of the moving volume; \bar{u}_i and \bar{T} , values of these functions in the surrounding fluid; R , mole radius;

$$A_1 = (b_1 + b_2) \frac{\nu}{R}, \quad A_2 = b_3 \frac{k}{R} + b_4 \frac{\nu}{R}, \quad 2R = \beta L, \quad b_3 = b_1 \left(\frac{k}{\nu} \right)^{0.33}.$$

In [1, 2] it was assumed that $b_1 = \text{const}$, $b_2 = \text{const}$. In [5] the computational scheme for the temperature field in flows with $Pr \gg 1$, the coefficients b_2 and b_4 were determined as a function of the local Reynolds number γ_* .

An approximate analytical solution of system (1.2) makes it possible to obtain velocity fluctuations u_i' and temperature fluctuations T' at the point M_0 as the mole from the neighborhood of the point M passes through it:

$$u_i'(M_0) = V_s'(M) f_0(ps) \cos(s, x_i) + [\bar{u}_i(M) - \bar{u}_i(M_0)] f_1(ps); \quad (1.3)$$

$$T'(M_0) = [\bar{T}(M) - \bar{T}(M_0)] f_1(\lambda ps), \quad (1.4)$$

where $s = |MM_0|$, $f_0(x) = e^{-|x|}$, $f_1(x) = \frac{1}{|x|} (1 - e^{-|x|})$, $\hat{p} = 3A_1/RV_s'$, $\lambda = A_2/A_1$.

Turbulent shear stresses and heat flux at the point M_0 are obtained in the form of corresponding volume integrals of the expressions $F_{ik}(M, M_0) = -u_i' u_k'$ and $E_i(M, M_0) = -u_i' T'$ with weighting function $\varphi(M, M_0)$:

$$-\overline{\rho u_i' u_k'} = \rho \int_D F_{ik}(M, M_0) \varphi(M, M_0) d\tau,$$

$$-c \overline{\rho u_i' T'} = c \rho \int_D E_i(M, M_0) \varphi(M, M_0) d\tau.$$

After neglecting small terms (see [2, 4]), the expression for the turbulent shear stress-tensor component $-\overline{\rho u_i' u_j'}$ and the components of turbulent heat flux $-\overline{c \rho u_i' T'}$ are obtained in the form

$$-u_i' u_i' = -P_{ii} + 2\varepsilon_{ii}^m \frac{\partial \bar{u}_i}{\partial x_i}, \quad (1.5)$$

$$-u_i' u_k' = \varepsilon_{ii}^m \frac{\partial \bar{u}_k}{\partial x_i} + \varepsilon_{kk}^m \frac{\partial \bar{u}_i}{\partial x_k} \quad (i \neq k); \quad (1.6)$$

$$-u_i' T' = \varepsilon_{ii}^h \frac{\partial \bar{T}}{\partial x_i}, \quad (1.7)$$

where

$$P_{ii}(M_0) = \int_D V_s'^2 f_0^2(ps) \varphi(M, M_0) \cos^2(s, x_i) d\tau; \quad (1.8)$$

$$\varepsilon_{ii}^m(M_0) = \int_D V_s' f_0(ps) f_1(ps) \varphi(M, M_0) \cos^2(s, x_i) d\tau; \quad (1.9)$$

$$\varepsilon_{ii}^h(M_0) = \int_D V_s' f_0(ps) f_1(\lambda ps) s \varphi(M, M_0) \cos^2(s, x_i) d\tau. \quad (1.10)$$

In order to make the numerical solution of the heat and momentum transfer problem effective, it is possible to simplify the expressions for P_{ii} , ε_{ii}^m , and ε_{ii}^h in Eqs. (1.8), (1.9), and (1.10) using Eq. (1.1) to integrals along the line parallel to the axis x_j or to the local formulas (see [2, 5]).

The model [1, 2] with [4] was used to compute steady velocity and temperature fields in different channel flows (see [6-8] and others).

Figure 1 shows an example of the computed temperature nonuniformity Θ along the perimeter of a heat-emitting rod in a triangular grid of closely packed rods for different ratios of thermal conductivities of the rod and the fluid and Re (from [7]). Here $\Theta = \lambda_f(T - T_0)/QR^2$; λ_r , λ_f are the thermal conductivities of the rod and the fluid; Q is the characteristic volumetric heat generation in the rod; R is the rod radius; 1-4 are for $\lambda_r/\lambda_f = 0.27, 0.68, 1.69, 6.35$; 1'-4' are experimental data from [9]. Similar results make it possible to obtain an understanding of overheating and thermal stresses in heat-generating elements.

Computations of velocity fields in the hydrodynamic stabilization region in fluid flows in circular and plane gaps were carried out in [10] using the approximations (1.5)-(1.7). It was mentioned there that the computed development of turbulent flow qualitatively agrees with the existing experimental data. However, a more rapid stabilization in the entrance region is observed in computations than in the experiment. Computed radial shear stress distributions $u'v'$ at different transverse sections of the flow in a circular pipe are shown in Fig. 2 (Re = 285,000). Here 1-3 represent $x/R = 8.28, 41.4, \text{ and } 190$; y is the distance from the pipe wall; \bullet, \circ, \square are experimental results from [11] for the same sections.

Thus, approximations of turbulent shear stresses (1.5), (1.6) with (1.1) which are well justified to compute steady flows, may have some error in the central part of the flow with hydrodynamic stabilization (curve 2 in Fig. 2).

The point here is, apparently, the turbulence intensity in the moving part of the fluid depends strongly on its history, i.e., in the hypothesis for the fluctuating energy of the mole it is necessary to include convective terms of turbulent energy in the moving fluid.

Many studies have appeared in recent years in which the models for turbulent transfer are based on equations for turbulent energy (see [12-15] etc.). Such models, in principle, should better describe the hydrodynamic stabilization region than the three-dimensional model [1, 2]. However, such models in the existing literature are applicable basically only to two-dimensional flows. Hence, in order to retain the advantages of the approximations for turbulent shear stresses (1.5), (1.6) and take into account certain advantages of the approach from [12-15] and others, it is necessary to combine ideas used in the models of the type [1, 2] and models with turbulent energy conservation equation.

In the following development of the model for turbulent transfer [1, 2], Prandtl's hypothesis (1.1) for fluctuating velocity of the mole $V_s^i(M)$ is replaced by a certain approximate equation representing the equation for the fluctuation energy balance. This equation along with hypothesis for the directed turbulent length scale L_s can also be used to determine velocity fluctuations of the moles $V_s^i(M)$ generated in the flow.

An algorithm for the approximation of conservation equation and its application in three-dimensional model was formulated by Buleev, verification of the model and the determination of empirical constants were carried out by Zinin.

2. Approximation of the Conservation Equation for Fluctuating Energy. The equation for turbulent energy is expressed in the form

$$q^2 \equiv \overline{u_1'^2} + \overline{u_2'^2} + \overline{u_3'^2}$$

which is written in the form

$$\frac{\partial q^2}{\partial t} + \sum_{k=1}^3 \overline{u_k} \frac{\partial q^2}{\partial x_k} = 2\nu \left| \frac{\partial V}{\partial n} \right|^2 - \sum_{i,h=1}^3 \overline{2u_i' u_h'} \frac{\partial u_i}{\partial x_h} - 2\nu \sum_{i,k=1}^3 \overline{\left(\frac{\partial u_i}{\partial x_k} \right)^2} + \sum_{k=1}^3 \frac{\partial}{\partial x_k} \left[\nu \frac{\partial q^2}{\partial x_k} - \frac{2}{\rho} \overline{u_k p'} - \sum_{i=1}^3 \overline{u_k' u_i' u_i'} \right]. \quad (2.1)$$

The first two terms on the right-hand side of Eq. (2.1) describe the generation of fluctuating energy Γ_1 and Γ_2 due to the dissipation of kinetic energy of the basic flow, the third term describes dissipation of the fluctuating energy, and the last three terms denote diffusion of fluctuating energy and pressure energy.

The expression $\Gamma_2 = -2 \sum_{i,h=1}^3 \overline{u'_k u'_i \frac{\partial u'_i}{\partial x_h}}$ is obtained from Eqs. (1.5) and (1.6) assuming that $P_{ii} = P$, $\varepsilon_{ii}^m = \varepsilon$. As a result we have

$$\Gamma_2 = 2\varepsilon |\partial V / \partial n|^2.$$

It is seen that the expression Γ_2 is similar in structure to the production term Γ_1 .

Following the traditional manner (see [12, 13]), the dissipation s is approximated by the expression

$$s = -2\nu \sum_{i,h=1}^3 \overline{\left(\frac{\partial u'_i}{\partial x_h}\right)^2} = -\frac{c^2}{L^2} (\nu + \varepsilon) q^2,$$

where c^2 is an empirical constant which is close to unity or a weak function of the local Reynolds number γ_* .

Pressure fluctuations p' in the expression $-\overline{u'_i p'}$, as also u' and T' will be coupled with the passage of moles through the neighborhood of the given point which, to a certain extent, also conveys the excess pressure within itself.

If the total static pressure in the turbulent flow is expressed in the form $p + \rho P$, where $P = (1/3)(P_{11} + P_{22} + P_{33})$, then, in accordance with the qualitative model of turbulent transfer of scalar conservative characteristic we take

$$-\overline{u'_h p'} = \tilde{\varepsilon}_{hk} \left(\frac{\partial p}{\partial x_h} + \rho \frac{\partial P}{\partial x_h} \right) = \tilde{\varepsilon}_{hk} \frac{\partial p}{\partial x_h} + \frac{1}{3} \rho \tilde{\varepsilon}_{hk} \frac{\partial q^2}{\partial x_h}.$$

Here $\tilde{\varepsilon}_{kk}$ are positive coefficients. Put $\tilde{\varepsilon}_{kk} = \theta \varepsilon_{kk}$, $0 < \theta < 1$, then

$$-\frac{1}{\rho} \overline{u'_h p'} = \theta \varepsilon_{hk} \frac{1}{\rho} \frac{\partial p}{\partial x_h} + \frac{\theta}{3} \varepsilon_{hk} \frac{\partial q^2}{\partial x_h}.$$

Experimental studies on the structure of turbulence carried out by different authors do not establish the practical role of the term $(\partial/\partial x_k) \theta \varepsilon_{kk} (\partial p/\partial x_k)$ in Eq. (2.1) and hence it is neglected now.

The third moment $\overline{u'_k u'_i u'_i}$ for the fluctuations u'_i and u'_k at the point M_0 will be directly computed using the same three-dimensional model [1, 2]. We get from Eq. (1.3), retaining the major terms

$$\overline{u'_i u'_i} (M_0) \approx V'^2 (M) \cos^2(s, x_i) f_0^2; \quad (2.2)$$

$$-\sum_{i=1}^3 \overline{u'_h u'_i u'_i} = -\int_D V'^3 (M) f_0^3 \varphi(M, M_0) \cos(s, x_h) d\tau. \quad (2.3)$$

The expression within the integral sign in Eq. (2.3) is a sign-variable function. Hence, there is a logical basis to neglect the terms with third moments in Eq. (2.1). Thanks to the simplicity of the terms of the type (2.3), we replace now derivatives $(\partial/\partial x_k) [V'^3 f_0^3]$ or $\zeta \varepsilon_{kk}^m \times (\partial q^2/\partial x_k)$ and combine them with approximations for $-\overline{u'_k p'}$. We assume

$$-\frac{1}{\rho} \overline{u'_h p'} - \sum_{i=1}^3 \overline{u'_h u'_i u'_i} \approx \tilde{\theta} \varepsilon_{hk} \frac{\partial q^2}{\partial x_h},$$

where $\tilde{\theta} = \theta/3 + \zeta$.

Finally, in the case of stationary field of the mean flow it is necessary to put $\partial q^2/\partial t = 0$ in Eq. (2.1).

The semiempirical approximation for the stationary turbulent energy equation (2.1) is finally written in the Cartesian coordinate system:

$$\frac{L^2}{\nu} \sum_{h=1}^3 u_h \frac{\partial q^2}{\partial x_h} + c^2 \left(1 + \frac{\varepsilon}{\nu}\right) q^2 - L^2 \sum_{h=1}^3 \frac{\partial}{\partial x_h} \left(1 + \tilde{\theta} \frac{\varepsilon_{hk}}{\nu}\right) \frac{\partial q^2}{\partial x_h} = 2 \left(1 + \frac{\varepsilon}{\nu}\right) L^2 \left|\frac{\partial V}{\partial n}\right|^2. \quad (2.4)$$

Since the model (1.5)-(1.10) does not require the time averaged mean kinetic energy q^2 of the fluctuating motion in the single neighborhood of the point M but requires the initial velocity V' of moles moving from the neighborhood of the point M , it is necessary to obtain a relation connecting $V'(M)$ and $q^2(M)$. The expression for $\overline{u'_i{}^2}$ from Eq. (1.5) is used for this purpose.

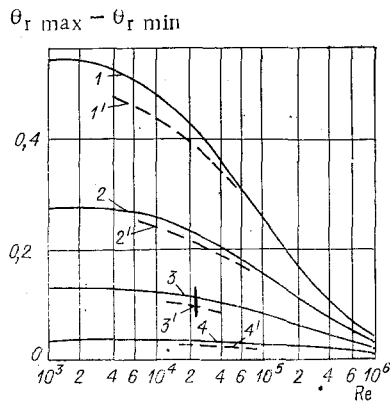


Fig. 1

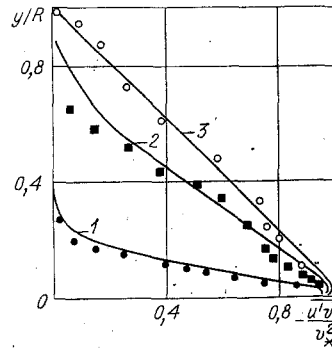


Fig. 2

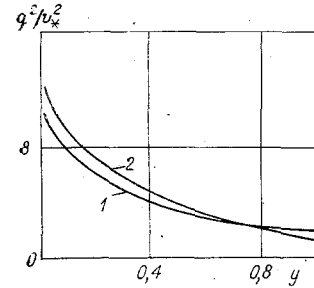


Fig. 3

Summing Eq. (1.5) with respect to the index i , taking into account (1.8), and assuming $\varepsilon_{11}^m = \varepsilon_{22}^m = \varepsilon_{33}^m$ we get approximately

$$q^2(M) \approx \int_D V'^2 f_0^2(p_s) \varphi(M', M) d\tau. \quad (2.5)$$

Equation (2.5) is rewritten in the form

$$q^2(M) \approx \kappa(\gamma_*) V'^2(M) \quad \text{or} \quad V'^2(M) \approx \frac{1}{\kappa} q^2(M), \quad (2.6)$$

where $\kappa(\gamma_*)$ is an empirical function close to $f_0^2(a_i \xi_i)$.

Anisotropy in the mole displacement velocity in different directions is described by the equation

$$V_i'^2 = \frac{L_i^2}{\bar{L}^2} V'^2, \quad \text{where} \quad \bar{L}^2 = \frac{1}{3} \sum_{j=1}^3 L_j^2. \quad (2.7)$$

The expressions for (1.5)-(1.10) for P_{ii} , ε_{ii}^m , ε_{ii}^h can be written in the following form, using Eqs. (2.4) and (2.6) after simplifying them to integrals along the line parallel to the axis x_i :

$$P_{ii}(M_0) = \frac{1}{3} \int_{-1}^1 V_i'^2 f_0^2(a_i \xi_i) H(\xi_i) d\xi_i; \quad (2.8)$$

$$\varepsilon_{ii}^m(M_0) = c_1 L_{i0} \int_{-1}^1 \frac{1}{\mu} V_i' f_0(a_i \xi_i) f_1(a_i \xi_i) G(\xi_i) d\xi_i; \quad (2.9)$$

$$\varepsilon_{ii}^h(M_0) = c_1 L_{i0} \int_{-1}^1 \frac{1}{\mu} V_i' f_0(a_i \xi_i) f_1(\lambda a_i \xi_i) G(\xi_i) d\xi_i, \quad (2.10)$$

where

$$\begin{aligned} \xi_i &= \frac{x_i - x_{i0}}{\alpha L_{i0}}, \quad a_i = \alpha L_{i0} \hat{p}_i, \quad \hat{p}_i = \frac{3A_1}{RV_i'}, \\ H(\xi) &= 6|\xi|^2(1 - |\xi|), \quad G(\xi) = |\xi|^3(1 - |\xi|), \\ \lambda &= \frac{b_1 \left(\frac{k}{v}\right)^{0.67} + b_2}{b_1 + b_2}, \quad c_1 = \frac{1}{5} \mu \alpha = 0.18. \end{aligned}$$

The coefficient μ taken from scheme (1.1) is introduced here to keep the continuity of the algorithm. The coefficient $a_i = \alpha L_{i0} \hat{p}_i$ present in the argument of functions f_0 and f_1 is written in the form

$$a_i = \frac{c_2}{\gamma_*} \frac{L_{i0}}{L_i}, \quad (2.11)$$

where

$$c_2 = \frac{12}{\mu \alpha} \left(\frac{\alpha}{\beta}\right)^2 (b_1 + b_2), \quad \tilde{\gamma}_* = \frac{1}{\mu} \frac{L^2 v}{L}. \quad (2.12)$$

Arguments of the functions f_0 and f_1 can be computed as in [4], i.e., according to (1.1) assume

$$\tilde{\gamma}_* = \gamma_* = \frac{L^2}{v} \left| \frac{\partial V}{\partial n} \right|.$$

It is not worth replacing only $\tilde{\gamma}_*$ by γ_* in the central part of the flow where the quantity $\partial V/\partial n$ is small.

It is important to mention here that if only production and dissipation terms are retained in Eq. (2.4), it will almost identically reduce to the hypothesis for fluctuating velocity of the mole (1.1), i.e., the hypothesis (1.1) can be considered the simplest form of turbulent energy conservation equation. The quantities μ and c^2 in Eqs. (1.1) and (2.4), respectively, are related by the expression $\mu^2 \approx 2/c^2$.

Equation (2.4) for the given coefficients and the right-hand side is solved numerically by finite-difference method.

After the numerical solution of Eqs. (2.4) and (2.6), the resulting values of $V'(M)$ are used to compute normal stresses and coefficients ε_{ii}^m and ε_{ii}^h using Eqs. (1.9) and (1.10). Momentum equations are solved after this. In view of the nonlinearity of momentum equations and Eq. (2.4) the hydrodynamic problem is solved by successive approximations.

For massive computations of velocity and temperature fields in different channels Eqs. (2.9) and (2.10) for ε_{ii}^m and ε_{ii}^h can be simplified locally

$$\varepsilon_{ii}^m = c_1 f_0(\eta) f_1(\eta) \frac{L_i^2 V'}{\mu v L}, \quad c_1 = 0.18; \quad (2.13)$$

$$\varepsilon_{ii}^h = c_1 f_0(\eta) f_1(\lambda \eta) \frac{L_i^2 V'}{\mu v L}, \quad \eta = \frac{65}{\tilde{\gamma}_*}. \quad (2.14)$$

3. Computation of Steady Velocity Field in a Circular Pipe. The values of empirical coefficients c , $\tilde{\theta}$, and $\kappa(\gamma_*)$ in Eqs. (2.4) and (2.6) were determined in the computation of velocity field in turbulent flow in a circular pipe.

The momentum equation for steady mean flow in a circular pipe has the form

$$-\frac{1}{r} \frac{\partial}{\partial r} r (v + \varepsilon_{ii}^m) \frac{\partial w}{\partial r} = \frac{1}{\rho} \frac{\partial p}{\partial z}. \quad (3.1)$$

Introducing nondimensional variables

$$\xi = \frac{r}{a}, \quad U = \frac{w}{v_*}, \quad \Phi = \frac{a v_*}{v}, \quad \text{Re} = \frac{2a\bar{w}}{v} = 2\bar{U}\Phi, \quad (3.2)$$

where a is the radius of the pipe, $v_*^2 = \frac{a}{2} \frac{1}{\rho} \left| \frac{\partial p}{\partial z} \right|$, Eq. (3.2) is rewritten in the following manner:

$$-\frac{1}{\xi} \frac{\partial}{\partial \xi} \xi \left(1 + \frac{\varepsilon_{rr}^m}{v} \right) \frac{\partial U}{\partial \xi} = 2\Phi.$$

The equation for the conservation of turbulent energy for such a flow along with the equation for fluctuating velocity of the mole has the form

$$c^2 \left(1 + \frac{\varepsilon_m}{v} \right) q^2 - L^2 \frac{1}{r} \frac{\partial}{\partial r} r \left(1 + \tilde{\theta} \frac{\varepsilon_{rr}}{v} \right) \frac{\partial q^2}{\partial r} = 2 \left(1 + \frac{\varepsilon_m}{v} \right) L^2 \left| \frac{\partial V}{\partial n} \right|^2, \quad (3.3)$$

$$V_i^2 = 3L_i^2 \left(\sum_{i=1}^3 L_i^2 \right)^{-1} \frac{q^2}{\kappa(\gamma_*)}.$$

In the computations carried out by the authors the factor $(1 + \varepsilon_m/v)$ in the terms describing the production and dissipation of fluctuating energy was approximated by the relation

$$1 + \frac{\varepsilon_m}{v} = 1 + 0.2 \frac{L^2}{v} \left| \frac{\partial V}{\partial n} \right|.$$

The coefficient ε_{rr}/v in Eq. (3.3) was computed from

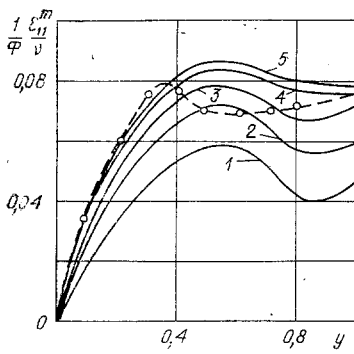


Fig. 4

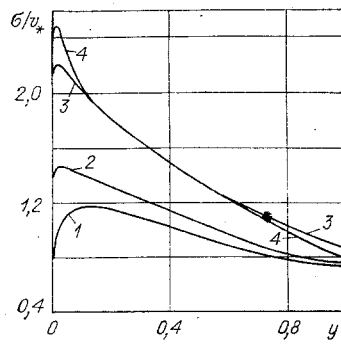


Fig. 5

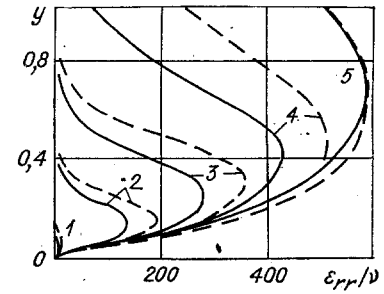


Fig. 6

$$\frac{\epsilon_{rr}}{\nu} = 0.18 f_0(\eta) f_1(\eta) \frac{L^2}{\nu} \left| \frac{\partial V}{\partial n} \right|_*$$

where $\eta = 65/\gamma_*$.

Preliminary solution of Eq. (3.1) along with Eqs. (3.3) and (2.9) showed that in order to obtain satisfactory agreement with experimental data of the computed fluctuating energy fields q^2 , the fields of averaged velocity and averaged velocity fluctuations, it is necessary to give the following values to the empirical coefficients and functions $\kappa(\gamma_*)$:

$$c^2 = 1.1 + 7/\sqrt{\gamma_*}, \quad \alpha = 0.78, \quad \mu = 1.15.$$

$$\left(\frac{\alpha}{\beta}\right)^2 b_1 = 1.6, \quad \left(\frac{\alpha}{\beta}\right)^2 b_2 = 6.4, \quad \tilde{\theta} = 0.1, \quad \kappa = 0.2 + 0.8 \exp(-50/\gamma_*^{0.75}). \dagger$$

Figure 3 shows the solution for q^2 in the flow through a circular pipe (curve 2). The curve 1 represents experimental data of Laufer [17] ($Re = 5 \cdot 10^5$).

Computed profiles of eddy viscosity coefficient ϵ_{11}^m/ν and rms fluctuations σ_1/v_* and σ_3/v_* in the flow through a circular pipe are shown in Figs. 4 and 5. The curves 1-5 in Fig. 4 are the computed radial distribution of viscosity coefficient for $\Phi = 200, 500, 1000, 5000,$ and $20,000$, respectively ($Re = 6000, 17,000, 38,000, 230,000,$ and 1 million), \circ is the eddy viscosity profile computed from Laufer's data [17] ($\Phi = 10,000$). Curves 1 and 3 in Fig. 5 are the experimental values of the rms fluctuations of the velocity σ_1/v_* , σ_3/v_* [17], the curves 2 and 4 denote the corresponding computed values of rms fluctuations ($Re = 500,000$).

4. Computation of the Hydrodynamic Stabilization Region. Computations of velocity fields in the initial segments of the circular pipe and the plane gap were carried using the numerical scheme for two-dimensional flow described in [10]. Coefficients ϵ_{11}^m/ν were computed using Eq. (2.3), taking into account Eqs. (2.4) and (2.6). Figure 6 shows computed profiles of eddy viscosity ϵ_{rr}^m/ν at different sections of the pipe at $Re = 300,000$. The results obtained from Eq. (1.1) is compared with the computed results. The dashed lines in Fig. 6 indicate ϵ_{rr}^m/ν obtained from Eq. (1.1), the continuous lines are obtained from Eq. (2.4). The curves 1-5 correspond to sections $x/R = 0.38, 6.85, 17.3, 37.7,$ and 140 ($Re = 300,000$).

It is seen from Fig. 6 that when Eq. (1.1) is replaced by Eq. (2.4), the computed coefficients ϵ_{rr}^m/ν are appreciably reduced in the central part of the entrance region of the flow field which leads, respectively, to a longer stable region when compared to the computations from [10].

A weakly expressed local maximum of the longitudinal velocity component has appeared on the axis of the pipe at a distance of about 20 diameters from the entrance, which is established by experiments but was not observed in earlier computations [10].

5. Application of the Model. On the basis of the generality of the governing equations and the hypotheses used here and also from preliminary computations of the velocity and temperature fields using Eqs. (1.5)-(1.7) and (2.4)-(2.12), it is possible to state that the given variant of the model for turbulent transfer in principle makes it possible to compute velocity and temperature fields in the hydrodynamic and thermal stabilization zones in channels of arbitrary form. However, in order to compute jets or external flows it is necessary to improve the equations for turbulent length scales L and L_i .

\dagger The difference in the values of empirical coefficients from the corresponding values given in [16] is due to the choice of $\psi(M, M_0)$.

From the point of view of the approximations used in the present paper for the turbulent energy equation (2.1), the system of Eqs. (1.5)-(1.7) and (2.4)-(2.12) can be considered as a three-dimensional model for turbulent transfer [1, 2] "to the second-order approximation."

LITERATURE CITED

1. N. I. Buleev, "Theoretical model of turbulent transfer mechanism in fluid flows," in: Heat Transfer [in Russian], Izd. Akad. Nauk SSR, Moscow (1962).
2. N. I. Buleev, "Theoretical model for turbulent transfer in three-dimensional flow," in: Trans. III Geneva Conference on Peaceful Uses of Atomic Energy [in Russian], Vol. 8, New York (1968).
3. N. I. Buleev, "Experiment on the use of three-dimensional model for turbulent transfer," Teplofiz. Vys. Temp., 14, No. 2 (1976).
4. N. I. Buleev, "Further development of three-dimensional model for turbulent transfer in incompressible flows," in: Turbulent Wall Shear Layer [in Russian], Vol. 1, Inst. Teor. Fiz., Siberian Branch, Academy of Sciences of the USSR, Novosibirsk (1975).
5. N. I. Buleev and G. A. Zinina, "Correction to governing hypothesis for three-dimensional model of turbulent transfer," in: Problems in Atomic Physics and Technology [in Russian], Ser. Physics and Technology of Nuclear Reactors, Vol. 2, NIKIET, Moscow (1982).
6. N. I. Buleev and N. V. Gusev, "Computation of turbulent flows in eccentric circular gaps," in: Problems in Atomic Physics and Technology [in Russian], Ser. Reactor Design, Vol. 6, Obninsk (1977).
7. N. I. Buleev and R. Ya. Mironovich, "Heat transfer in longitudinal flow in triangular grid of rods," in: Problems in Atomic Physics and Technology [in Russian], Ser. Physics and Technology of Nuclear Reactors, Vol. 4, NIKIET, Moscow (1982).
8. N. I. Buleev and L. S. Sukhomlin, "Hydrodynamics and heat transfer in turbulent flows in concentric gaps," in: Problems in Atomic Physics and Technology [in Russian], Ser. Physics and Technology of Nuclear Reactors, Vol. 2, NIKIET, Moscow (1982).
9. P. A. Ushakov, A. V. Zhukov, and N. M. Matyukhin, "Azimuthal nonuniformity in temperature of fuel elements located in straight grids with turbulent flow of liquid metals," Teplofiz. Vys. Temp., 15, No. 4 (1977).
10. N. I. Buleev and G. A. Zinina, "Computation of the stabilization of turbulent flow in the entrance region of circular pipe," in: Problems in Viscous Flows [in Russian], Institut Teoreticheskoi i Prikladnoi Mekhaniki, Siberian Branch, Academy of Sciences of the USSR (1981).
11. L. P. Volnistova, B. N. Gabrionovich, Yu. D. Levchenko, and Yu. P. Trubakov, Turbulent Characteristics of Flow at the Entrance Region of Circular Pipe [in Russian], Preprint FÉI-725, Obninsk (1976).
12. A. N. Kolmogorov, "Equations of incompressible turbulent flow," Izv. Akad. Nauk SSSR, Ser. Fiz., 6, Nos. 1-2 (1942).
13. A. S. Ginevskii and V. A. Ioselevich, "Computational method for turbulent boundary layer," in: Mechanics of Fluids (Results of Science and Technology) [in Russian], Vol. 11, VINITI, Moscow (1978).
14. V. G. Lushik, A. A. Pavel'ev, and A. E. Yakubenko, "Three-parametric model for turbulent shear," Izv. Akad. Nauk SSSR, Mekh. Zhidk. Gaza, No. 3 (1978).
15. U. Frost and T. Moulden, Turbulence. Principles and Applications [Russian translation], Mir, Moscow (1980).
16. N. I. Buleev and G. A. Zinina, "Three-dimensional model for turbulent transfer using equating for fluctuating energy," Vestsi Akad. Nauk BSSR, Ser. Energ. Navuk, No. 3 (1983).
17. J. O. Hinze, Turbulence, McGraw-Hill, New York (1959).